Theory of the rotation of Janus and Epimetheus

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The Saturnian coorbital satellites Janus and Epimetheus present a unique dynamical configuration in the Solar System, because of high-amplitude horseshoe orbits, due to a mass ratio of order unity. As a consequence, they swap their orbits every 4 years, while their orbital periods is about 0.695 days. Recently, Tiscareno et al.(2009) got observational informations on the shapes and the rotational states of these satellites. In particular, they detected an offset in the expected equilibrium position of Janus, and a large libration of Epimetheus. We here propose to give a 3-dimensional theory of the rotation of these satellites in using these observed data, and to compare it to the observed rotations. We consider the two satellites as triaxial rigid bodies, and we perform numerical integrations of the system in assuming the free librations as damped. The periods of the 3 free librations we get, associated with the 3 dimensions, are respectively 1.267, 2.179 and 2.098 days for Janus, and 0.747, 1.804 and 5.542 days for Epimetheus. Our theory explains the amplitude of the librations of Janus and the error bars of the librations of Epimetheus, but not an observed offset in the orientation of Janus.

Context

The Saturnian satellites Janus and Epimetheus present the peculiar dynamical configuration to be locked in a $1:1$ mean motion resonance. This results in horseshoe orbits and in orbital swaps every 4 years, i.e. Janus and Epimetheus swap their semimajor axes (see Fig.1).

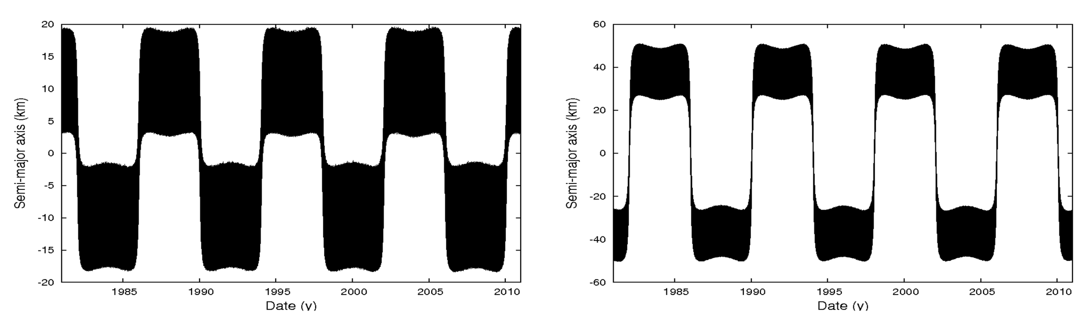

Figure 1: Variations of the semi-major axes of Janus and Epimetheus. We can see the 8-y-periodic orbital swaps.

Recently, Tiscareno et al. [3] derived the shapes and moments of inertia from Cassini data (see Tab.1), and parameters of their rotations as well. We here present a theoretical study of their rotations based on the moments of inertia derived by Tiscareno et al., and a comparison with the observed rotation.

Table 1: Moments of inertia, taken from Tiscareno et al.(2009). They have been derived from best-fit numerical shape models.

	A/MR^2	B/MR^2	C/MR^2	$(B-A)/C$
Janus	0.360	0.407	0.470	0.100 ± 0.012
Epimetheus	0.328	0.469	0.476	$0.296^{+0.019}_{-0.027}$

Method

We here assume the two satellites to be rigid bodies whose rotations are perturbed by Saturn. For each body, the Hamiltonian of the rotational dynamics reads:

$$\mathcal{H} = \underbrace{\frac{nP^2}{2} + \frac{n}{8}\left[4P - \xi_q^2 - \eta_q^2\right]\left[\frac{\gamma_1+\gamma_2}{1-\gamma_1-\gamma_2}\xi_q^2 + \frac{\gamma_1-\gamma_2}{1-\gamma_1+\gamma_2}\eta_q^2\right]}_{\text{Kinetic energy}} \underbrace{- \frac{3}{2n}\frac{\mathcal{G}M_{♄}}{d_{♄}^3}\left(1+\frac{5}{2}\left(\frac{\mathcal{R}_{♄}}{d_{♄}}\right)^2\right)\left[\gamma_1(x_{♄}^2+y_{♄}^2)+\gamma_2(x_{♄}^2-y_{♄}^2)\right]}_{\text{Saturnian perturbation}} \quad (1)$$

with the following canonical variables:

$$p = l+g+h \qquad P = \frac{G}{nC}$$
$$r = -h \qquad R = \frac{G-H}{nC} = P(1-\cos K) = 2P\sin^2\frac{K}{2}$$
$$\xi_q = \sqrt{\frac{2Q}{nC}}\sin q \qquad \eta_q = \sqrt{\frac{2Q}{nC}}\cos q$$

where n is the satellite's mean orbital motion, $q=-l$, and $Q = G-L = G(1-\cos J) = 2G\sin^2\frac{J}{2}$. The coefficients of the Hamiltonian are defined as follows:

$$\gamma_1 = J_2\frac{MR^2}{C} \qquad \gamma_2 = 2C_{22}\frac{MR^2}{C}$$

and the angles can be seen on the figure above, reproduced from (Henrard [1]). They use 2 sets of Euler angles, the first one (h,K,g) locates the position of the angular momentum in the first frame $(\vec{e_1},\vec{e_2},\vec{e_3})$, while the second (g,J,l) locates the body frame $(\vec{f_1},\vec{f_2},\vec{f_3})$ in the second frame tied to the angular momentum.

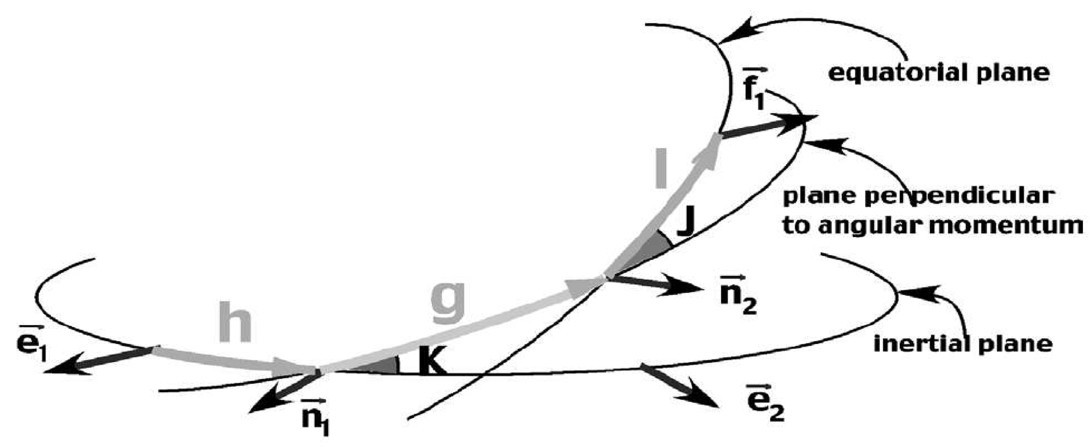

Figure 2: The angles (reproduced from Henrard [1]).

x and y are the first two coordinates of the center of the perturber (here Saturn) in the frame $(\vec{f_1},\vec{f_2},\vec{f_3})$ bound to the satellite. We get its coordinates from the JPL HORIZONS ephemerides.
After some canonical transformations introducing the slow angles, we find the equilibrium of the system, confirming the synchronous rotation and giving the obliquity of the body. Then, a centering of the Hamiltonian on the equilibrium and a conversion into polar coordinates yields:

$$\mathcal{H}(u,v,w,U,V,W) = \underbrace{\omega_u U + \omega_v V + \omega_w W}_{\text{3-d oscillator}} + \underbrace{\mathcal{P}(u,v,w,U,V,W)}_{\text{Perturbation}}, \quad (2)$$

where (u,v,w) are angles, (U,V,W) the actions associated, and $(\omega_u,\omega_v,\omega_w)$ the 3 frequencies of the small oscillations about the equilibrium. These oscillations are expected to be damped, but knowing them gives information on the response of the system to the perturbation. The equations derived from the Hamiltonian are numerically integrated, and an algorithm a frequency analysis is applied on the solutions to extract the different periodic contributions.

Results

The Tab.2 gives the values of the 3 fundamental frequencies, numerically determined.

Table 2: Periods of the free librations, determined numerically.

	Janus	Epimetheus
T_u	1.26713 d	0.74717 d
T_v	2.17884 d	1.80386 d
T_w	2.09798 d	5.54234 d

In order to deliver theories of rotation that can be easily compared with observations, we express our results in the following variables :

- Longitudinal librations γ
- Latitudinal librations
- Orbital obliquity ϵ

The longitudinal librations used here are the physical librations, defined by the librations about the synchronous rotation, i.e. $p - < n > t$. Because of the orbital swaps, we considered these librations far for the swaps, for instance over the time spans [1998.5 : 2001.7] and [2002.5 : 2005.7] (Fig.3).

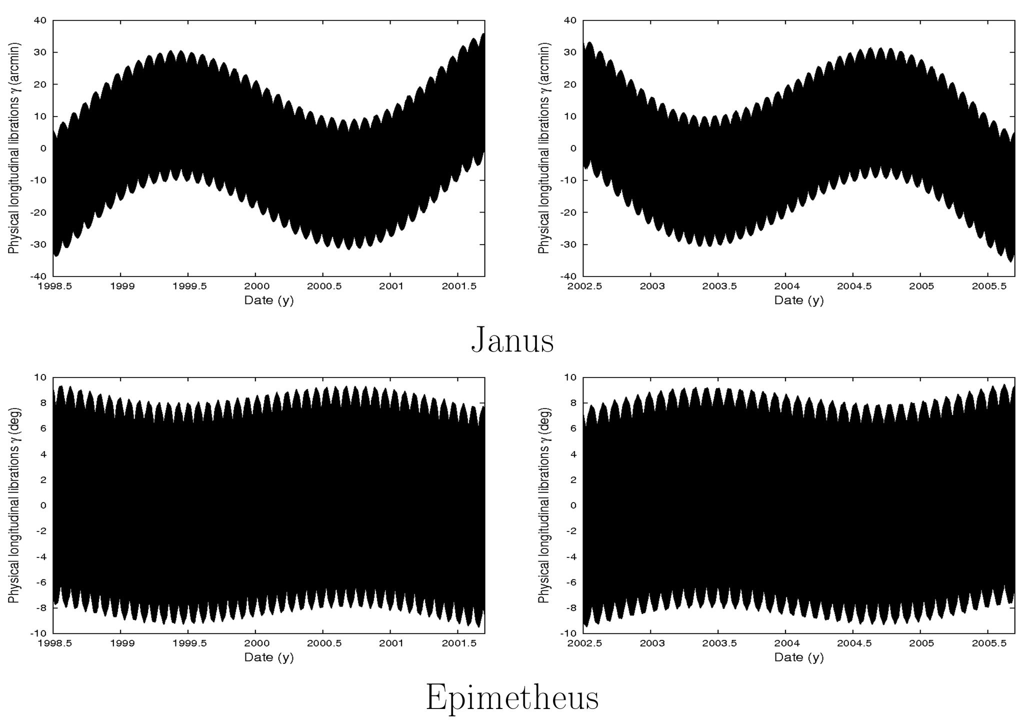

Figure 3: Physical longitudinal librations γ of Janus (up) and Epimetheus (down). These librations have been obtained in removing a fitted mean spin from the orientation of the longest axis on the inertial reference frame, respectively 3304.356301 rad/year (left) and 3303.673150 rad/year (right) for Janus, and 3302.7839016 and 3305.24602 rad/year for Epimetheus.

We can see on this figure variations of the amplitude of the librations, that can induce uncertainties on the observed librations. The next table gives our theoretical results, that we compare with Cassini's observations.

Table 3: Comparison between the observations of Tisacereno et al. (2009) and our results. The column "our simulations" refers to the results of the numerical integration while mean values refers to a study of the uncertainties.

	Tiscareno et al. (2009)	Our simulations	Mean values
Janus			
Librations γ	$-0.3 \pm 0.9°$	$-0.34 \pm 0.03°$	$-0.36 \pm 0.06°$
Obliquity ϵ	–	5.95 arcsec	6.72 ± 0.82 arcsec
Longitudinal offset	$5.2 \pm 1°$	–	0
Latitudinal offset	$2.3 \pm 1°$	–	0
Epimetheus			
Librations γ	$-5.9 \pm 1.2°$	$-8.6 \pm 0.9°$	$-9.9^{+4.9}_{-13.4}$ °
Obliquity ϵ	–	10.83 arcsec	–
Longitudinal offset	$< 1°$	–	0
Latitudinal offset	$< 1°$	–	0

Conclusion

Our model, based on a complete dynamical modelisation of the orbits of Janus and Epimetheus, can explain the uncertainties given by the observations of the longitudinal librations of the satellites. Moreover, it shows that neglecting the obliquity of these bodies is a good approximation. However, it cannot explain the observed offsets of the orientation of Janus. This could show that considering these bodies as rigid homogeneous ellipsoids is irrelevant.

References

References

[1] Henrard J., 2005, *The rotation of Io*, Icarus, 178, 144-153

[2] Noyelles B., 2010, *Theory of the rotation of Janus and Epimetheus*, Icarus, in press

[3] Tiscareno M.S., Thomas P.C. & Burns J.A., 2009, *The rotation of Janus and Epimetheus*, Icarus, 204, 254-261